AD-769 155

A DEVELOPMENT OF DETERMINISTIC AND STOCHASTIC TRANSFER PRICING MODELS AND THEIR APPLICATION TO DOD

James D. Bain

Air Force Institute of Technology
Wright-Patterson Air Force Base, Ohio

7 May 1973

DISTRIBUTED BY:

NTIS
National Technical Information Service
U. S. DEPARTMENT OF COMMERCE
5285 Port Royal Road, Springfield Va. 22151

UNCLASSIFIED

Security Classification

AD769155

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)

AFIT/ENS
Wright-Patterson AFB, Ohio

2a. REPORT SECURITY CLASSIFICATION

Unclassified

2b. GROUP

3. REPORT TITLE

A DEVELOPMENT OF DETERMINISTIC AND STOCHASTIC TRANSFER PRICING MODELS AND THEIR APPLICATION TO DOD

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

MASTER'S DEGREE THESIS

5. AUTHOR(S) (First name, middle initial, last name)

JAMES D. BAIN, CAPTAIN, USAF

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
7 MAY 1973	~~65~~ 75	28

8a. CONTRACT OR GRANT NO.

b. PROJECT NO.

c.

d.

9a. ORIGINATOR'S REPORT NUMBER(S)

GSA/SM/73-1

9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. DISTRIBUTION STATEMENT

Distribution of this document is unlimited

Approved for public release; IAW AFR 190-17

JERRY C. HIX, Captain, USAF
Director of Information

12. SPONSORING MILITARY ACTIVITY

13. ABSTRACT

Attempts to initiate user funding at Arnold Engineering and Development Center (AEDC) in the 1968-1971 time period were very unsuccessful. At that time much of the failure was attributed to the uncertainty of demand for the facility's services.

This thesis first examines the origination of profit centers and transfer pricing in the private sector of the economy and the manner in which these concepts precipitated user funding in DOD. Next it presents mathematical formulations of optimal transfer prices under static and deterministic market conditions. Conditions of stochastic market demand and cost are then applied to those formulations to provide optimal transfer prices using maximization of expected profit as the decision criterion.

Finally the stochastic demand model is applied to a simplified Test and Evaluation environment in order to determine the form of the optimal transfer price at AEDC. Consideration is also given to the conditions surrounding use of transfer prices to improve efficiency [illegible] in the [illegible] sector of the econo[illegible]

DD FORM 1 NOV 65 1473

1a

UNCLASSIFIED

Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
TRANSFER PRICING						
TRANSFER PRICING MODELS						
DETERMINISTIC/STOCHASTIC TRANSFER PRICING MODELS						
USER FUNDING						
SERVICE FUNDING						
REVOLVING FUNDS						
ARNOLD ENGINEERING AND DEVELOPMENT CENTER						
ENZER TRANSFER PRICING MODEL						
HIRSHLEIFER TRANSFER PRICING MODEL						
PROFIT CENTERS						

A DEVELOPMENT OF DETERMINISTIC AND STOCHASTIC TRANSFER PRICING MODELS AND THEIR APPLICATION TO DOD

THESIS

GSA/SM/73-1

James D. Bain
Captain USAF

GSA/SM/73-1

A DEVELOPMENT OF DETERMINISTIC AND STOCHASTIC TRANSFER PRICING MODELS AND THEIR APPLICATION TO DOD

THESIS

Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

Air University

in Partial Fulfillment of the

Requirements for the Degree of

Master of Science

by

James D. Bain, B.S. Engineering Science
Captain USAF

Graduate Systems Analysis

June 1973

Approved for public release; distribution unlimited.

Preface

This thesis presents the results of a study on transfer pricing and its applications in the private and public sectors of the economy. The findings will hopefully prove of some benefit in the search for ways to increase the efficiency of resource allocation processes through internal pricing mechanisms.

Many individuals have contributed to make this effort possible. Foremost on this list is Dr. Herman Enzer. As an instructor in economic analysis courses, he has provided me with insights and experiences in systems analysis that will always bear upon my analytical reasoning. As a thesis advisor, he has provided me with guidance, patience, and wisdom that have made this study possible. Special thanks also go to my reader, Professor Joseph P. Cain, who gave much of his time in the reading and analysis of drafts. In spite of their beneficial assistance, however, I alone bear responsibility for any errors herein. Statements referencing material from the various sources are my interpretations of the original text.

Finally, I must gratefully thank my wife, Brenda, and daughters, Jamie and Dara, who have contributed and sacrificed much to make this thesis and the AFIT experience possible and worthwhile. To them I dedicate this study.

James D. Bain

Contents

List of Figures

Abstract

Attempts to initiate user funding at Arnold Engineering and Development Center (AEDC) in the 1968-1971 time period were very unsuccessful. At that time much of the failure was attributed to the uncertainty of demand for the facility's services.

This thesis first examines the origination of profit centers and transfer pricing in the private sector of the economy and the manner in which these concepts precipitated user funding in DOD. Next it presents mathematical formulations of optimal transfer prices under static and deterministic market conditions. Conditions of stochastic market demand and cost are then applied to those formulations to provide optimal transfer prices using maximization of expected profit as the decision criterion.

Finally the stochastic demand model is applied to a simplified Test and Evaluation environment in order to determine the form of the optimal transfer price at AEDC. Consideration is also given to the conditions surrounding use of transfer prices to improve efficiency and resource allocation in the government sector of the economy.

I. Introduction

Centralization and Decentralization

The classic businessman's view of organization is based on his attitude toward the complexities of coordination. The result is often a vertical, hierarchial business structure in which orders emanating from above are executed by those working below. To ensure that the lower echelons carry out their assigned directives and optimize the firm's objective functions, top managements have devised various means of maintaining organizational control (Ref 4:399). One of the most authoritarian of these is known as centralization. In addition to providing a concentration of control not otherwise possible, centralization is usually accompanied by some of the advantages of size. Principal among these are diversification of risk, centralized financial structures, and management specialization, particularly in the planning and advisory functions (Ref 26:781). Centralized managements are further characterized by a complete separation of the persons responsible for budgeting and resource allocation and those responsible for operational activities (Ref 7:352).

In order for top management to be able to issue directives based on knowledge of the many interrelated decision variables, it is necessary that they have access to all pertinent information available through the lower echelons. Not all decisions can be made by top management as it is obvious that there will be some loss and misinterpretation in the information assimilation and transmission process. Therefore, some decisions will have to be delegated to the lower echelons. Although these difficulties have been somewhat alleviated

by the advent of the high speed computer, information remains a costly requirement of centralization.

Relevant information is not the only problem in centralized organizations. Criticism is frequently based on the dissatisfactions of lower echelon management. Motivational problems frequently arise whenever it is necessary to obtain high level approval of solutions to low level problems. Other unsatisfactory aspects of centralization frequently cited in the management literature are the inability to provide a platform for change, the lack of continuity in the event of a change in the central figure, and a tendency to be sluggish and hidebound by rules and regulations (Ref 21:18).

Because of these and other problems with centralized organizational frameworks, many companies have restructured along lines of decentralization. One of the more important influences in this direction has been recognition that the man on the spot knows more about his own sphere of activity than does the corporate management. Given appropriate authority the lower level manager can act quickly and flexibly in response to local conditions. In addition to removing some of the sheer weight of decision making from higher echelons, this provides a valuable training ground for junior managers.

One of the principle motivations for decentralizing in most companies is the expectation that profitability can be increased by giving direct profit incentives to more people in management. Theoretically, decentralization provides greater possibilities for lower echelon management initiative and thereby increases the probability of improved resource utilization. But for any subunit to be an economically efficient operation its behavior must reflect the

same profit motivation as that of an independent company. This can and does create problems. It is possible that this independence will permit one or more of the divisions to increase its own profit through actions that will decrease profit for the firm as a whole. The result is a control problem for the decentralized firm. To be effective in this situation a control system must motivate each manager to act in ways that optimize company objectives, provide a basis for evaluating managerial performance, and provide relevant information at points where decisions are to be made (Ref 13:145).

Some generalized characteristics of decentralized organizations are separate statements of income and expenditure for the various sectors, the delegation of authority to lower echelon managers to make decisions which will affect their sectors profit, and managerial performance evaluations in terms of items within their control. It is important to note, however, that the notions of centralization and decentralization are relative and not absolute. In a lengthy study Zannetos has shown it is impossible to make unqualified generalizations as to whether a company is centralized or decentralized (Ref 28:B-54). The notion of decentralization, however, is clearly more appropriate under circumstances where the subunits have operations that are closely akin to those of independent operating companies. But boundary lines in any organization can become a significant obstacle. In any economic structure one of the major coordinating devices between entities is the price system; this is particularly true for the competitive type of market (Ref 4:403).

The Role of Price Theory

Economists have long been fascinated with the operation of the competitive market price system as an instrument for control of economic activity. Self interest theoretically inspires each entrepreneur to seek the most profitable uses of his resources and to employ the factors of production, adopt the technology, and produce the items that will provide society with the most efficient allocation of resources. If a different resource allocation becomes desirable, factor and product prices will change to move marginal resources from areas of excess supply to areas of excess demand (Ref 15:2). The market test of efficiency will enable the firm capable of the most effective use of resources to pay a higher price for them and thus draw them away from less productive uses. In such instances the measure of economic worth is the market price (Ref 26:818).

Vertical and horizontal integration are frequently the result of efforts to reduce costs by merging overhead, transportation, and the staff functions. When the resulting firm decentralizes and there is an exchange of goods and services between different divisions, the market test for economic efficiency is often lost. In the case of some products, market prices do not exist; in others, they may be available but their use might distort the savings which initiated the original integration. Hence, decentralized firms have a real problem in determining interdivisional exchange prices that will foster efficient resource utilization. One mechanism that has helped solve this problem is the profit center.

Profit Centers

In recent years control systems have focused on the responsibility center to motivate managers and to measure both their performance and that of their divisions. Responsibility centers are simply organizational units headed by responsible persons and usually exist in the form of expense, profit, or investment centers. The profit center has come to be regarded as the most efficient form of corporate organization, and it is now considered primarily responsible for approximately four-fifths of all major U. S. corporations being largely decentralized (Ref 8:29).

Under the profit center concept the company is broken down into manageable units which are treated as independent companies, each controlled by a manager who is responsible for making a profit and obtaining a reasonable return on investment from the assets at his disposal. Performance is then measured in terms of revenues and costs. Some units might be called profit centers because their managers have authority over selling costs, inventory levels, and product mixes, all of which are major determinants of profit.

The objectives of profit centers are as follows (Ref 26:781,785):

1. Provide a basis for delegating authority to those managers who have a close familiarity with individual products or markets.

2. Bring lower echelon managers into a more direct contact with the firm's profit objective.

3. Provide an integrated training ground for the junior managers.

4. Help appraise the financially measureable aspects of managerial performance in the divisions.

5. Provide a measure of the profitability of the resources invested in the divisions.

6. Guide division managers toward decisions that will increase company profits.

The greatest danger in the profit center concept has already been mentioned. It is the ever present possibility that a division is permitted or encouraged to take actions that will increase its profits while decreasing those of the firm. This is generally referred to as suboptimization and firms have attempted to gain control over it through constraints on authority and the use of pricing mechanisms.

Transfer Prices

The prices used to transfer products or services between responsibility centers within a company, as contrasted with market prices which measure the exchanges between a company and the outside world, are referred to as transfer prices (Ref 6:428). One of the most difficult problems of a decentralized organization is the determination of a sound, workable scheme of transfer prices. If the divisions buy all raw materials from outside the company and sell all outputs in the external market, internal profit measurements will be relatively simple and efficient resource allocation will be facilitated. When, however, there is an integration of two successive production processes or the absence of an external market, the market test for efficiency no longer exists. The challenge then becomes to establish transfer prices that will, among other things, direct the efficient allocation of resources and assist in the determination of each profit center's revenues and costs (Ref 26:817).

Transfer prices can serve their purpose only if they lead division managers to make the same decisions top management would make if it had full access to all the pertinent information and the time to study the

problems. The selection of the transfer price to be utilized depends on a variety of factors such as the kinds of information available and the objectives management hopes to accomplish via the profit center concept. Some of the more commonly used prices are variations of market price, negotiated price, full cost, marginal cost, variable cost, and costs derived through mathematical programming techniques. Although some degree of success has been achieved with most of these pricing rules there is no general agreement that any of them is economically correct. In spite of this, early successes by such corporate giants as General Motors, DuPont, and General Electric have had significant effects on other companies and the government (Ref 13:144).

II. Transfer Pricing within DOD

Revolving Funds

Long before the successful application of the profit center concept in private industry, economists and management scientists alike were interested in improving resource allocation within the Defense Department through some form of a competitive market price system. One proposal designed to accomplish this was offered by Abba Lerner in the early stages of World War II (Ref 18:221). He suggested decentralization of the military establishment and creation of markets for the purchase and sale of defense produced goods and services. He hoped that the reorganized defense structure would operate in a manner similar to that of the private, competitive sector of the American economy.

Lerner's system specified that subordinate commanders down the line should be allowed to make use of their specialized knowledge to accomplish their assigned missions and to obtain the necessary resources to do so. Once their objectives were made known, individual commanders would have to acquire the men and materiel to carry out their mission by purchasing them with limited funds in a competitive market system. The network of markets for men and materiel would provide the information which the training and materiel commands needed to determine the relative usefulness of various skills and instruments. Theoretically, at least, this model would tend to produce a more efficient utilization of resources (Ref 18:222).

For a variety of reasons Lerner's proposal was not accepted but it did have beneficial side effects. Of prime importance was the fact

that it spawned interest in the problem areas of efficiency and resource allocation in the military establishment. When private industry began having success with decentralization and profit centers, the ideas that Lerner had advocated received new attention. The eventual result was the introduction into DOD of what is now called the revolving fund concept.

The revolving fund is basically an instrument of decentralized decision making which creates buyer and seller arrangements between users and suppliers within DOD. The intent of these arrangements is to encourage efficient choices by the various military organizations in the selection of goods and services (Ref 7:343). Revolving fund arrangements are usually either in the form of stock funds or industrial funds, and are merely appropriated working capital arrangements designed to create a business atmosphere. Stock funds finance activities that purchase expendable materials from outside the government, such as enlisted men's clothing, petroleum products, and medical supplies. Industrial funds finance activities such as military air and sea transportation, testing services, and government printing establishments (Ref 18:224).

Hitch and McKean refer to the command organizations that utilize these funding arrangements as "firms," and they go on to say (Ref 18:225):

> The firms operating both kinds of funds purchase their inputs, adopt businesslike accounting systems, charge prices to the military units or others who buy their outputs, submit profit-and-loss statements, and try to maximize "profits." ... The funds are expected to bring about three types of improvement. First, their managers will be motivated to conduct them more efficiently, because there will be a more appropriate indicator of efficiency than would otherwise exist .... Second, customers will be motivated to find cheaper substitutes and

> ways of using less of these products .... Third, the outputs of these establishments will be allocated more efficiently--that is, put to their most valuable uses--because they will be rationed by "market prices" rather than by bare shelves or semi-arbitrary quotas.

One of the most important considerations in any price-market system is the measure of economic worth ascribed to each of the transferred commodities. It has been said that regardless of how management responsibilities are divided no decision maker can efficiently allocate resources if the prices do not reflect their full value (Ref 21:20). As long as military goods and services are not traded in the open market, full value prices cannot be determined. It does seem reasonable, however, that transfer prices can be established that will improve resource allocation within the DOD. That this is at least partially correct has been borne out by Navy experiences with industrial funding.

In addition to other areas, particular success has been claimed by the Navy in its Research and Development (R&D) facilities. The Naval Research Laboratory was placed on industrial funding in early 1953. This was quickly followed by implementation of the system at the Naval Ordnance Laboratory and the Naval Ship Research and Development Center. The results were such that the Navy added fourteen more R&D activities to the industrial funding system in the 60's and early 70's (Ref 2:3). Claims of success for the system were not limited to the Navy. The Army also implemented a form of industrial funding in some of its transportation depots, ordnance arsenals, and research facilities (Ref 10:472). Unfortunately, this pattern of success does not hold everywhere.

The AEDC Experience

There has at least been one major effort to apply the revolving fund concept where the general concensus is that results were unsatisfactory. Prior to discussing that specific situation a brief historical review of its implementation is in order.

In the early 1950's Congress passed a set of laws entitled The Budgeting and Accounting Procedures Act. The purpose of these laws was to provide the basic framework for budgeting and financial management procedures within the federal government. The Department of Defense and the Air Force in particular were slow to comply with these laws and subsequently came under attack from the Government Accounting Office and Congress.

In answer to these pressures the Assistant Secretary of Defense, Comptroller, issued a set of directives in 1966 under the collective title of the Resource Management System (RMS). Although designed to overhaul existing budgeting and accounting systems, a major feature of RMS was to make users of resources responsible for budgeting and paying for their needs (Ref 3:1).

Compliance with these directives for the Air Force resulted in implementation of the Air Force Industrial Fund (AFIF) activities which included the Airlift Service, Depot Maintenance, Laundry and Dry Cleaning, and Printing Funds. A modification of the industrial fund concept was outlined for use in Research and Development activities and designated as a service or user fund. Although its operation appeared fairly simple the desired results were not always achieved. It required managers of R&D projects to issue funded work orders to a service fund manager responsible for performing the needed test

activities. After completion of the task, the service fund manager would charge the costs to the service fund which in turn would receive reimbursement for the project from the manager of the project. At the end of each accounting period the service fund should show revenues equal to expenses. It was designed as a zero profit industrial fund, but it did not always operate that way.

DODI 7220.24 required that at least one R&D activity within each branch of the DOD be operated under the service fund concept or some similar alternative not later than 1 July 1968. To comply with this instruction, the Air Force selected the Arnold Engineering Development Center (AEDC) located in Tennessee. AEDC was selected because it was considered to be the only R&D activity which had a management information system capable of supporting service fund procedures. The management device responsible for this selection was the accounting system employed by ARO Inc., the contractor for Arnold's work. Plans were also made to eventually implement service funding at the Rome Air Development Center in New York and the Air Force Flight Test Center at Edwards AFB, California (Ref 1:2).

Service funding was initiated at AEDC on a trial basis in July of 1968. The costs recoverable from charges to users amounted to about 87 per cent of the facility's total expenses. In the first year AEDC was funded as it had been in prior years, but as the service fund manager it billed itself as program manager and associated the basic costs incurred with each of the benefitting programs on an after-the-fact basis. For FY 70 the user programs were funded to simulate actual service fund operations, but they were restricted from using them for any purpose other than AEDC testing without AFSC approval. In FY 71

this restriction was removed and the service fund was considered fully operational. Unfortunately, difficulties emerged early in the test period which persisted throughout the life of the fund at AEDC (Ref 1:7).

In December of 1969, General Lowe, Commander of AEDC, gave a briefing to the Air Staff and cited the major problems arising from the service fund concept. These included an inability of the Center to accurately predict workloads, an inability to reduce expenses when the forecast workloads failed to materialize, and competition from test facilities which were not under the service fund concept (Ref 1:9). As an example, the forecast tunnel time utilization attributable to specific projects in FY69 was 40,000 hours. Of those, only 16,000 hours actually materialized. To further confuse their forecasting capability, some 28,000 hours of tunnel testing not anticipated were requested by and provided to other-facility users (Ref 24:1).

As a result of these experiences there were many suggestions to eliminate user funding or implement some alternative funding system. One suggestion that appeared in March of 1971 proposed that all expenses arising from the fundamental existence of AEDC be institutionally funded while expenses directly associated with tests be recovered through the service fund. Perhaps the biggest advantage of this suggestion was that it would have permitted AEDC to lower the prices charged for services rendered. This plan was not implemented, however, because such a combined funding arrangement was considered to be in violation of existing DOD directives. After continued failures, the user funding concept was eventually terminated at AEDC with the conclusion of FY72.

The future of service funding in Air Force R&D facilities does not end with the AEDC misfortune. In October of 1971 Deputy Secretary of Defense Packard sent out a memorandum requesting formation of a Study Group to investigate the funding policies in existence at 26 major DOD Test and Evaluation (T&E) activities and to make recommendations for improvement (Ref 22:3). The Study Group was chaired by Deputy Assistant Secretary of Defense (Systems Policy and Information) George W. Bergquist, and the final report was submitted in April 1972 (Ref 9:1-102).

The study recognized the many complexities of financial management involved in a uniform system of funding, but stated that the primary problem was to reconcile the differing motivations of the T&E support activity managers and the project managers who used the T&E facilities (Ref 9:22). It is reasonable to extend this one step further and state that the objectives of both groups must be in accord with the objectives of the DOD. The user funding concept is a form of decentralized management and a primary requisite of this is that the objectives of the lower level managers should promote actions that are generally consistent with the objectives of the organization (Ref 24:18).

The Study Group established four alternatives as possibilities for dealing with the problems associated with T&E funding activities. These are as follows (Ref 9:75):

1. User funding of direct costs only at all 26 activities and identification of each activity as a separate line item in budget justification.

2. User funding of direct costs and some indirect costs at all 26 activities.

3. User funding of at least direct cost at all but six of the 26 ranges. Those six have long been designated as national ranges because of certain operational and administrative differences.

4. Reaffirm existing funding policies, which amounts to perpetuation of the many different existing arrangements.

Of these, the first was recommended, even though the Study Group recognized that this proposal constituted a combination of institutional and user funding which had previously been rejected as illegal. Consequently, they proposed a change in the law to make this alternative possible. The facts that all 26 T&E facilities would be affected and that users would be required to make adjustments in their long-range cost estimates of testing were viewed as the primary disadvantages of this alternative. In other words, the group viewed the changes as unsettling and felt that implementation would have to be carefully accomplished and closely monitored (Ref 9:75-77).

Historical Implications

A number of factors emerge from this historical background. Without denying the existence of others, a listing of some of the more important highlights follows:

1. There will be continued efforts to transfer successful private sector managerial philosophies and techniques to the public sector.

2. There will be continued efforts at decentralization which will involve some form of transfer pricing.

3. DOD will place an increasing emphasis on efficient resource allocation.

4. Some form of price-market system is fundamental to efficient resource allocation.

5. Objectives of lower level managers must be consistent with the objectives of the whole organization if decentralization is to be effective.

This thesis is a direct result of the above listed factors and the Air Force experience at AEDC. It is not an attempt to solve

single-handedly the problems of budgeting and financial management, but an effort to gain some insight into one aspect of motivating efficient resource allocation. The remainder of this thesis will therefore be concerned with economically defensible transfer pricing systems.

III. Deterministic Transfer Pricing Models

The Approach

In seeking reasons for the failure of user funding at AEDC, two questions seem to stand out. First, was the value established as the transfer price correctly chosen, and second, what effect does uncertainty have on a transfer price? Before addressing these questions it seems appropriate to recall that the profit center and transfer price concepts were initiated in DOD because of their success in private industry. The concepts were created under a viable market system and employed there prior to being transferred to the artificially induced market systems of the Defense Department.

The failure of the user funding systems at AEDC was preceded by similar misfortunes in industry. Such corporate giants as Ampex and Cutler-Hammer experienced near-disaster with their initial attempts at profit center reorganization (Ref 8:29). Successes and failures alike have occurred through the implementation of a variety of transfer pricing schemes. As mentioned previously, some commonly used values are variations of market price, negotiated price, full cost, marginal cost, and variable cost. It has been stated that the major prerequisite of a transfer price is that it be consistent with the objectives of the company (Ref 12:172), but it has also been argued that many of the systems employed are economically indefensible in both theory and practice (Ref 11:74).

If, in fact, many of the transfer prices employed in the private sector are economically indefensible, it seems premature to discuss the "correct" transfer price for use in the artificial markets of DOD.

Consequently, the approach of this study will be to postulate a viable market economy in the quest for an appropriate transfer price.

In spite of the inability to find complete agreement on a transfer price value, most economists concur that a model developed by Jack Hirshleifer has a reasonably sound economic basis. His model first appeared in an article published in 1956 under the title of "On The Economics of Transfer Pricing" (Ref 17:172-184). Since then it has frequently been used as a platform for theoretical and practical extensions.

The Hirshleifer Analysis

The Hirshleifer analysis assumes that a firm is composed of two or more autonomous profit centers, each of which has the objective of profit maximization, and that it is possible for the exchange of goods to take place between the centers. The problem is to determine the price at which the transfers are to take place, and the goal is to set the price such that the separate centers will make decisions which yield the largest aggregate profit for the firm as a whole.

His model assumes a firm with two such centers: a manufacturing division (seller) and a distribution division (buyer), where the commodity exchanged is referred to as the intermediate product. Both divisions exist under conditions of demand and technological independence where demand independence means that an additional sale by either division does not reduce the external demand for the other division. Similarly, technological independence means that operating costs of the divisions are independent of one another.

A number of cases with this basic setup were examined by Hirshleifer and the following conclusions were reached (Ref 17:183).

1. In the absence of an intermediate market, the optimal transfer price is the marginal cost of the selling division.

2. If the intermediate product has a perfectly competitive market, the transfer price should be the market price. If the intermediate market is imperfectly competitive, the transfer price should again be the marginal cost of the selling division.

3. In the absence of demand independence, the transfer price should be somewhere between marginal cost and market price.

4. The optimal transfer price rule leads to correct output decisions only where marginal rules apply. For capital decisions such as make or buy, examination of the total magnitudes involved is necessary.

The arguments employed by Hirshleifer in reaching these conclusions were based on economic theory and the use of graphical analysis. Many economists have attempted to fault his reasoning but the most valid criticisms have come from Shillingham, Gordon, and Enzer (Ref 26:827); (Ref 15:23); (Ref 14:7). These criticisms can be summarized as follows:

1. Most firms are unable to develop the required marginal cost and marginal revenue schedules necessary to implement the model.

2. Divisional autonomy is sacrificed through the necessity to introduce constraints specifically designed to prevent suboptimization.

3. In the absence of an external market the first division may be forced to operate at a loss. This fails to provide adequate consideration of the performance measurement concept of decentralization.

A recent analysis conducted by Herman Enzer dealt with these flaws, provided results that differ from those obtained by Hirshleifer, and is amenable to the introduction of uncertainty (Ref 14:1-58).

<u>The Enzer Analysis</u>

The basic assumptions of the Hirshleifer and Enzer analyses are the same except that the former relied on a graphical analysis while

the latter employed a mathematical analysis. Enzer's model postulated economic revenues and costs for the firm and its divisions and employed calculus to determine a set of optimizing conditions. By solving for the firm's optimal conditions first, he found it possible for the firm to designate to the divisions the value the transfer price should take in order to maximize the firm's profits as a whole. Under a variety of static and deterministic conditions that value proved to be some form of an average cost.

The Enzer model will be presented with two cases. The first case will assume no market for the intermediate product and the second will assume the existence of such a market. In this model and all subsequent mathematical formulations the functions presented will be considered to be both continuous and differentiable.

Case 1: No Market for the Intermediate Product

The firm is made up of two divisions: Division 1 is a basic manufacturing center and Division 2 is a finishing and sales center. Both divisions are to be treated as autonomous profit centers, and along with the firm, have profit maximization as their objective.

Division 1 uses x_1 and x_2 as inputs to produce the intermediate output u where the input-output relationship can be expressed by the production function $u = f(x_1,x_2)$. The division purchases its inputs at known constant prices of r_1 and r_2 resulting in a total cost function of the form $C_1(u) = r_1x_1 + r_2x_2$. It sells its output u at the yet to be determined transfer price of r_u to Division 2. Letting r_uu represent revenue and Π_1 represent profit, Division 1's objective

may be written as

$$\text{maximize} \quad \Pi_1 = r_u u - r_1 x_1 - r_2 x_2$$

$$\text{subject to} \quad u = f(x_1, x_2).$$

The constraint is merely saying that the division wants to produce only what it will sell. Division 1's decision variables are x_1 and x_2.

Division 2 uses x_3 and u as inputs to produce the final product where the input-output relationship can be expressed as the production function $q = g(x_3, u)$. Total cost of production is represented by $C_2(q) = r_3 x_3 + r_u u$ where r_3 is the constant price of x_3, and r_u represents the unknown transfer price of u. The final product is sold on the market at the price p which produces a revenue denoted by pq. Representing its profit by Π_2, Division 2's objective may be written as

$$\text{maximize} \quad \Pi_2 = pq - r_3 x_3 - r_u u$$

$$\text{subject to} \quad q = g(x_3, u).$$

As before the constraint merely says the division wants to produce only what it can sell. For Division 2 the decision variables are x_3 and u.

For the firm as a whole revenues are obtained through the sale of q in the market, represented by pq, while total cost is $C(q) = r_1 x_1 + r_2 x_2 + r_3 x_3$. Denoting the firm's profit by Π, the objective may be written as

$$\text{maximize} \quad \Pi = pq - r_1 x_1 - r_2 x_2 - r_3 x_3$$

$$\text{subject to} \quad u = f(x_1, x_2) \text{ and } q = g(x_3, u).$$

The transfer price r_u does not appear in the equation of the firm because as a cost it is included in the term $r_1x_1 + r_2x_2$, and it is not a revenue because it constitutes a sale by the firm to itself. Decision variables for the firm as a whole are x_1, x_2, x_3, and u.

The above objective and constraint equations may be written in Lagrangian form for both divisions and the firm as

$$L_1 = r_u u - r_1x_1 - r_2x_2 - \lambda_1 \left[f(x_1,x_2) - u\right]$$

$$L_2 = pg(x_3,u) - r_3x_3 - r_u u$$

$$L = pg(x_3,u) - r_1x_1 - r_2x_2 - r_3x_3 - \lambda \left[f(x_1,x_2) - u\right]$$

where λ_1 and λ represent Lagrangian multipliers. Optimal solutions for these equations can be found by differentiation with respect to the decision variables and equating the differentials to zero. This results in the following set of first order equations necessary to produce a maximum:

<u>Div 1</u>	<u>Div 2</u>	<u>Firm</u>
$r_1 = -\lambda_1 f_1$		$r_1 = -\lambda f_1$
$r_2 = -\lambda_1 f_2$		$r_2 = -\lambda f_2$
$f = u$		$f = u$
	$pg_3 = r_3$	$pg_3 = r_3$
$\partial(r_u u)/\partial u = -\lambda_1$	$\partial(r_u u)/\partial u = pg_u$	$pg_u = -\lambda$

Here $f_1 = \partial f/\partial x_1, g_u = \partial g/\partial u$, etc. Note that u is determined for Division 1 by Division 2, and hence is a constant for Division 1. For Division 2 it is a variable and it is necessary to assume that r_u is a function of u. Hirshleifer's failure to consider the transfer price as a function of output is the primary difference between the

Enzer and Hirshleifer models. If r_u is assumed constant, the final solution reduces to that obtained by Hirshleifer.

To produce an optimal solution for the separate divisions and simultaneously produce an optimal solution for the firm, there must be agreement between the above sets of equations. Note that if the first equation for Division 1 is multiplied by dx_1, the second by dx_2, and the two equations are added, the result is

$$r_1dx_1 + r_2dx_2 = -\lambda_1 \ (f_1dx_1 + f_2dx_2).$$

The quantity to the left is the total differential of the cost equation for Division 1, and the quantity in parentheses on the right is the total differential of the production function. Therefore, the Lagrangian multiplier equals (-1) times the marginal cost of Division 1, i.e., $\lambda_1 = -dC_1/du$.

To force the three sets of equations to be consistent it is necessary to equate

$$\partial(r_u u)/\partial u = -\lambda_1 = pg_u = dC_1/du$$

This is accomplished easily enough, and to solve for an optimal value of the transfer price, let

$$r_u u = \int[dC_1(u)/du]du + k = C_1(u) + k$$

or $$r_u = [C_1(u) + k]/u = C_1(u)/u + k/u$$

where k represents the constant of integration, and the term $C_1(u)/u$ represents the average variable cost of Division 1. As k is arbitrary the selection of k = 0 would result in the transfer price being

equal to the average variable cost while setting k equal to total fixed cost would result in the transfer price being equal to average total cost. Hence, in this situation it is some sort of an average, and not necessarily marginal cost as Hirshleifer concluded. It is worthwhile to note here that by setting k equal to total fixed cost, Division 1's receipts just cover costs, i.e., $r_u u = C_1(u)$ + fixed cost. As k is purely arbitrary and there does not appear to be a rationale for a particular value, Division 1 can be treated as a cost center rather than a profit center by setting $k = 0$. This would alleviate the problem of negative profits that was created in the Hirshleifer model.

Case 2: Market for the Intermediate Product

The rationale and terms of this model are very similar to those of the preceding case except that it is now assumed that the intermediate product has an external market and that external demands for the intermediate and final products are independent. Hence, the product u is now $u = u_1 + u_2$ where u_1 represents that portion sold on the external market at the prevailing market price m, and u_2 represents that portion transferred to Division 2 at the yet to be determined transfer price of r_u. As Division 1 derives an external revenue, it is now properly referred to as a profit center. Using the same symbols employed in the preceding case with the noted addition, the corresponding objective and constraint equations become:

Division 1: maximize $\Pi_1 = mu_1 + r_u u_2 - r_1 x_1 - r_2 x_2$

subject to $u = f(x_1, x_2)$ and $u = u_1 + u_2$,

with decision variables x_1, x_2, u_1, u_2, and u;

Division 2: maximize $\Pi_2 = pq - r_3x_3 - r_uu_2$

subject to $q = g(x_3,u)$,

with decision variables x_3 and u_2;

Firm: maximize $\Pi = pq + mu_1 - r_1x_1 - r_2x_2 - r_3x_3$

subject to $u = f(x_1,x_2)$, $u = u_1 + u_2$, and $q = g(x_3,u)$,

with decision variables x_1, x_2, x_3, u_1, and u.

The corresponding Lagrangian functions become

$$L_1 = mu_1 + r_uu_2 - r_1x_1 - r_2x_2 - \lambda_1[f(x_1,x_2) - u] - \theta_1[u_1 + u_2 - u]$$

$$L_2 = pg\ (x_3,u_2) - r_3x_2 - r_uu_2$$

$$L = pg\ (x_3,u_2) + mu_1 - r_1x_1 - r_2x_2 - r_3x_3 - \lambda[f(x_1x_2)-u]-\theta[u_1+u_2-u]$$

where λ_1, λ, θ_1, and θ represent Lagrangian multipliers. The first order conditions to produce a maximum are then given by:

Division 1	Division 2	Firm
$r_1 = -\lambda_1 f_1$		$r_1 = -\lambda f_1$
$r_2 = -\lambda_1 f_2$		$r_2 = -\lambda f_2$
$f = u$		$f = u$
$u_1 + u_2 = u$		$u_1 + u_2 = u$
$\lambda_1 = -\theta_1$		$\lambda = -\theta$
$m = \theta_1$		$m = \theta$
	$pg_3 = r_3$	$pg_3 = r_3$
$\partial(r_uu_2)/\partial u_2 = \theta_1$	$\partial(r_uu_2)/\partial u_2 = pg_{u_2}$	$\theta = pg_{u_2}$

As in Case 1 it can be shown that $\lambda = \lambda_1 = -dC_1/du$, or λ_1 is (-1) times the marginal cost of Division 1. To produce simultaneously optimal

conditions it is necessary to set $\partial(r_u u_2)/\partial u_2 = m = -\lambda_1$, which leads to

$$r_u u_2 = \int m du_2 = \int \frac{dC_1}{du} du_2$$

$$= mu_2 + k = C_1(u_1 + u_2) + k$$

$$r_u = m + \frac{k}{u_2} = \frac{C_1(u_1 + u_2) + k}{u_2} .$$

The term k represents the constant of integration from the indefinite integrals. If k is set equal to zero and the market price is constant, the resulting optimal transfer price is equal to the market price as Hirshleifer theorized.

But if the market price of the intermediate product is affected by the quantity of production as in the case of imperfect competition, the middle integration is no longer valid as a market price interpretation and the optimal transfer price is given by the right hand side of the equality. In attempting to establish a value for the integration constant k, note that the numerator of the term $C_1(u_1 + u_2)/u_2$ represents the cost of producing both u_1 and u_2 and not just the amount purchased by Division 2. Consequently, a likely candidate for the constant is $-C(u_1)$ which would tend to make Division 2's costs more directly related to its own production efforts. Regardless of the value k is given, the transfer price will be an average cost and not marginal cost as Hirshleifer theorized.

The Revised Enzer Model

The Enzer models concluded that in the case of deterministic external market demands, the optimal transfer price should be some form of an average cost with the integration constant specifying what

the exact form will be. It is possible to introduce uncertainty into these models but the resulting equations become quite complicated. Consequently, some simplifying techniques will now be introduced.

The previous models have comprised a single stage solution to the problems of optimal input mixes and optimal output quantities. This will now be altered so that the problem will consist of two stages. The first stage is the determination of the optimal input mix, which will be assumed solved, and the second stage is the determination of the optimal output quantities, which is the problem at hand. The revised formulation will now be presented for the external intermediate market case only. A slight modification of this model is necessary to formulate the case of no external intermediate market.

As in the model of Case 2, Division 1 produces the intermediate product u. The output u is equal to the sum of u_1 and u_2 where u_1 represents the quantity of u to be sold on the external market at the price m and u_2 represents the quantity to be sold to Division 2 at the yet to be determined transfer price of r_u. The cost of producing u is given by $C_1(u) = C_1(u_1 + u_2)$. The division has a decision variable of u_1 and the objective of maximizing its profit which is given by

$$\Pi_1 = mu_1 + r_u u_2 - C_1(u_1 + u_2).$$

Division 2 produces the final product q for sale in the external market at the market price p. The product q is produced from inputs of u_2 plus a combination of other materials. The cost of u_2 is the yet to be determined transfer price r_u and the cost of all other inputs is given by $C_2(q)$. This division has q as a decision variable

and its objective is to maximize

$$\Pi_2 = pq - C_2(q) - r_u u_2.$$

The firm as a whole sells q and u_1 on the external market at the prices of p and m respectively. Its total costs are given by $C_1(u_1 + u_2)$, which is the cost of producing u, plus $C_2(q)$, which is the cost of all inputs to q other than u. With decision variables of u_1 and q the firm seeks to maximize

$$\Pi = mu_1 + pq - C_1(u_1 + u_2) - C_2(q).$$

The first order conditions for optimization of these objective functions are given by the partial derivatives with respect to the decision variables. These are

$$\text{Division 1:} \quad \frac{\partial mu_1}{\partial u_1} - \frac{\partial C_1(u_1 + u_2)}{\partial(u_1 + u_2)} \cdot \frac{\partial(u_1 + u_2)}{\partial u_1} = 0$$

$$\text{Division 2:} \quad \frac{\partial pq}{\partial q} - \frac{\partial C_2(q)}{\partial q} - \frac{\partial(r_u u_2)}{\partial u_2} \cdot \frac{\partial u_2}{\partial q} = 0$$

$$\text{Firm:} \quad \frac{\partial mu_1}{\partial u_1} - \frac{\partial C_1(u_1 + u_2)}{\partial(u_1 + u_2)} \cdot \frac{\partial(u_1 + u_2)}{\partial u_1} = 0$$

$$\frac{\partial pq}{\partial q} - \frac{\partial C_2(q)}{\partial q} - \frac{\partial C_1(u_1 + u_2)}{\partial(u_1 + u_2)} \cdot \frac{\partial(u_1 + u_2)}{\partial u_2} \cdot \frac{\partial u_2}{\partial q} = 0$$

To ensure optimal conditions in the firm and its divisions, the following equalities must hold

$$\frac{\partial(r_u u_2)}{\partial u_2} \cdot \frac{\partial u_2}{\partial q} = \frac{\partial C_1(u_1 + u_2)}{\partial u_2} \cdot \frac{\partial u_2}{\partial q}$$

$$\frac{\partial mu_1}{\partial u_1} = \frac{\partial C_1(u_1 + u_2)}{\partial u_1} .$$

Hence

$$r_u u_2 = \int m\,du_2 = \int \frac{dC_1(u_1 + u_2)}{du}\,du_2$$

$$r_u = m + \frac{k}{u_2} = \frac{C_1(u_1 + u_2) + k}{u_2}\,.$$

Note here that as in the Enzer model the middle terms in the equality hold only if the market price of the intermediate product is constant. This result is identical to that obtained in Case 2, and it is this last model that will be used as a basis for the introduction of uncertainty in the next chapter. Primary emphasis will be placed on application of the techniques that have evolved in stochastic inventory theory (Ref 16:370-396); (Ref 5: 110-153).

IV. Stochastic Transfer Pricing Models

The Approach

Selman and Selman (Ref 25:1-16) have summarized some of the decision rule alternatives available for use under conditions of certainty, risk, and uncertainty. By their classification certainty or deterministic situations are those that typically occur in the bulk of formal economic and management science theory. Usually the decision rule employed is to select that alternative from the set of all possibilities which maximizes (or minimizes) some given index such as value or utility. The deterministic models of the preceding chapter can be cited as examples of decision making under conditions of certainty.

Risk has been defined by those authors as that situation in which each action may result in more than one consequence depending upon the state of nature, and where each state of nature has a known or presumed known probability associated with it. The decision rules usually applied to conditions of risk are as follows:

1. Maximum Expected Value Criterion
2. Maximum Subjective Utility Criterion
3. Most Probable Future Principle Criterion
4. Expectation-Variance Principle
5. Simon-March "Satisficing Hypothesis"
6. Bayes Decision Criterion

Finally, complete uncertainty is defined as that situation in which each action may result in more than one outcome, depending upon the state of nature, but where each state of nature has an unknown probability. The decision rules associated with uncertainty do not depend upon the assignment of probabilities to the various states,

and are consequently classified as non-stochastic decision rules. They are as follows:

1. LaPlace Criterion of Insufficient Reason
2. Wald "Sure Thing" Principle of Minimax (Maximin)
3. Maximax Criterion
4. Hurwicz Alpha-Criterion of Optimism/Pessimism
5. Savage Minimax Regret Criterion

It is not the intention of this study to elaborate on these decision rules; information on any of them can be found in the literature of risk analysis.

The conditions of the market in decentralized organizations can only infrequently be described as complete uncertainty according to Selman's definition. Market demands, supplies, costs, and prices usually have some assumed probability distribution associated with them. Consequently, the models to be developed in this section will be based on what Selman has defined as risk. In other words, the components of the market are assumed to be stochastic in that they come from known or presumed known probability distributions.

One difficulty in constructing models to deal with stochastic conditions is the necessity to account not only for the various probabilities involved, but to incorporate the decision maker's attitude toward risk. Individuals are frequently classified as risk-indifferent, risk, evasive, or risk preferent. A person who is risk indifferent has no preference between playing a game of chance or receiving the expected value of the game with certainty. Such an individual is sometimes referred to as being linear in risk. The risk evader is one who demands that the expected value of the risky alternative exceed that of the guaranteed payoff before he is willing to accept the former. Finally, one who is a risk taker is willing to accept the risky

alternative in lieu of a guaranteed payoff even if the expected value of the former is less than the value of the latter (Ref 19:346).

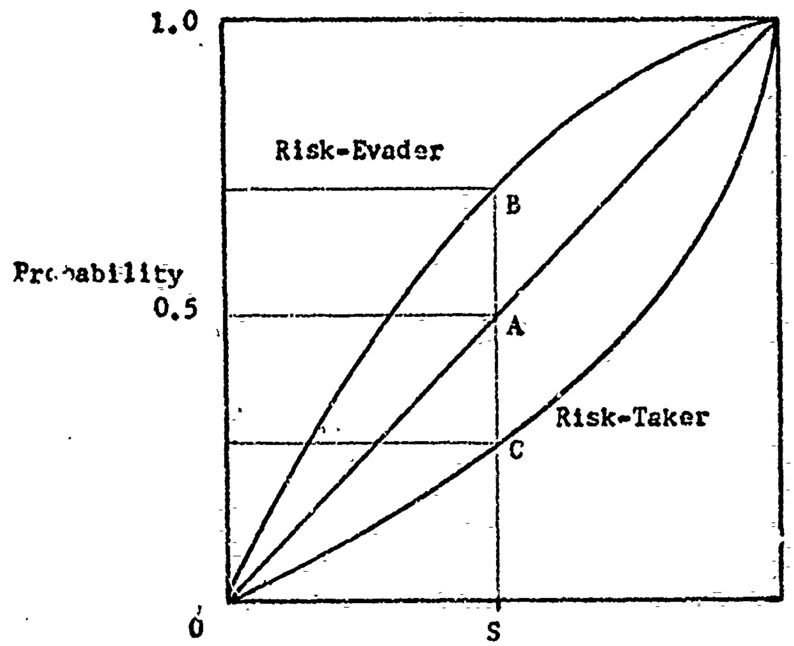

Fig. 1. Attitudes Toward Risk

Obviously, not all risk evaders will require the same probability of winning prior to accepting a game of chance, nor will all risk takers. Furthermore, even though two people have a similar risk attitude, they may select different alternatives in a given circumstance because of dissimilar probability assessments. Figure 1 graphically illustrates the varying attitudes toward risk. Suppose, for example, that the certainty equivalent represented by S is \$3 and the value in the game of chance is \$6. The individual who is linear in risk has no preference between accepting the \$3 as a sure thing and a 0.5 probability of winning \$6 in the game of chance. Point A represents such an

attitude toward risk. However, the risk evader represented by point B requires a probability of winning greater than 0.5 before he will forego the certainty equivalent. Finally, point C illustrates that the risk taker will accept the game with a probability of winning less than 0.5. It is worthwhile to note from this illustration that the person who is linear in risk will order his alternatives for a specific decision in accordance with expected value.

A secondary objective of this study is to analyze the differences between deterministic and stochastic transfer pricing rules. Of the deterministic models available, those presented in Chapter III appear most valid; and of the decision rules available for situations of risk, expected value is most amenable. Consequently, the stochastic models to be presented will be based on the expected value criterion, and it will be assumed that the decision makers in question are risk indifferent.

The Expected Value Criterion

It is sometimes argued that maximization of expected value is not always a valid decision criterion. This is in fact the case, but there are many practical business situations in which expected monetary value will produce desirable results. Furthermore, there are many situations in which it may not correspond to the decision maker's objectives, but constraints can be included that will make it more acceptable. Consider the following example: Suppose that the probability density function is known to be skewed either to the left or right, such as in Figure 2. Although the weighted probabilities in these figures are the same on each side of the expected values, the possible range of

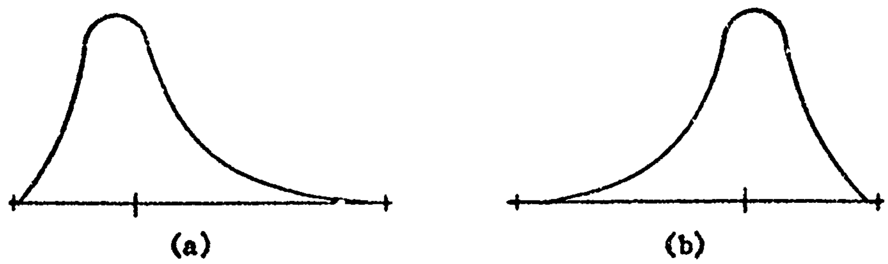

Fig. 2. Skewed Probability Densities

numbers to be taken by the random variable is larger on one side than the other in both of the above figures. As such, expected value in either of these two cases may be inconsistent with the behavior pattern of some particular entrepreneur.

More specifically, suppose that the probability density function for the sale of newspapers is similar to that in Figure 2(a). The markup for newspapers is not large and the salvage value of the unsold quantity is very small compared to the cost of procurement. Consequently, a particular vendor may rationally decide to purchase fewer than the expected value of demand with the attitude that it is better to be unable to satisfy demand than to have unsold newspapers. A realistic method in which the expected value does become a valid guide to action in this situation is through the introduction of a goodwill cost. If the vendor is forced to deal with the opportunity cost of customers who may never return, his decisions will become closer aligned with the expected monetary value criterion.

Similarly, a demand density skewed to the right as in Figure 2(b) might cause an entrepreneur to order or produce more than the expected

value. In this case zero salvage value or a holding cost would serve as a mechanism to penalize him for guessing and bring his decision criterion closer to that of expected value.

Schlaifer has provided a formal rule as a guide for the reasonable man to use in deciding whether or not expected monetary value is an appropriate decision criterion (Ref 23:29).

> Test for the Validity of Expected Monetary Value as a Guide to Action: Expected monetary value should be used as the decision criterion in any real decision problem, however complex, if the person responsible for the decision would use it as his criterion in choosing between (1) an act which is certain to result in receipt or payment of a definite amount of cash and (2) an act which will result in either the best or the worst of all possible consequences of the real decision problem.

The intuitive proof of this rule is too lengthy for presentation here, but the interested reader can find it in Schlaifer's text (Ref 23: 29-30, 43-44). As an example of its application, however, consider a businessman who can decide that expected value is commensurate with his wishes in a situation where the worst possible loss and best possible profit are given by \$W and \$B respectively. According to Schlaifer's rule, then, expected value is a valid decision criterion for all alternatives whose outcomes fall between these two reference points. Furthermore, a person who values alternatives at their expected monetary value can always find a utility scale such that the utility of every consequence between these reference points is numerically equal to the monetary value of that act. In other words the expected utility of any act involving consequences within this range is numerically equal to the expected monetary value of the act (Ref 26:44).

The value of this decision rule lies in its application to delegation of authority. A compelling motivation in the development of optimal transfer pricing rules is to enable a second party to determine the choice preferred by another. If it were possible for subordinates to act in a manner commensurate with top managements' objectives, then delegation of authority would become more desirable. Schlaifer's rule for use of expected value purports to accomplish this end. First, the person who is ultimately responsible for a certain class of decisions does not have to specifically consider each action, but rather provides reference points for a range over which a specific decision rule is to be considered valid. To facilitate this, there are many business situations in which it is possible for the responsible person to delegate his evaluations of probabilities associated with various events by designating consideration of certain past actions in conjunction with one or more of the many statistical techniques available. Secondly, a top manager who has delegated decisions that are to him somewhat routine becomes free to concentrate on those problems in which expected losses or gains are too great for him to justify the expected value criterion. In the transfer pricing situation, the upper and lower bounds on the expected value criterion can occur through the application of specific decision constraints.

The Basic Model

Throughout this section derivations will concentrate only on the first order conditions for optimization; second order conditions are assumed satisfied. Initially, market demand is assumed to be a random variable from some known or presumed known probability distribution.

Other market parameters are assumed deterministic. Items are produced for a single period at specific costs, but prior to the beginning of a planning period the entrepreneur must make an irrevocable decision regarding his output quantity without certain knowledge of what value the random variable demand will take. To say that the output decision is irrevocable implies that for the given production period the firm must commit itself to order certain input quantities.

Assume initially that any revenue potential from production in excess of demand and output insufficient to meet demand is lost. In other words it is not possible to backlog sales and excess production cannot be stored because the commodity is perishable. The last deterministic model presented in Chapter III which was based upon a firm with two divisions and an external intermediate market is the basis for the stochastic extension that follows.

The market demand for the intermediate product is represented by the random variable μ which is always greater than zero and has some known demand distribution represented by $g(\mu)$. Division 1 must make a decision as to the quantity of u_1 it will produce to satisfy this uncertain demand. Once that commitment has been made costs are known and it is possible to represent the form the division's profit will take. Revenue from sales of u_1 on the external market will be given by

$$R(u) = \begin{Bmatrix} m\mu \text{ for } \mu < u_1 \\ mu_1 \text{ for } \mu \geq u_1 \end{Bmatrix} .$$

The first segment represents the total revenue received from the sale of u_1 when no more units are demanded at the market price m than have been produced. The second segment represents the total revenue when

at least as many units are demanded as have been produced. Revenue from the sale of u_2 to Division 2 will be fixed at $r_u u_2$ once the transfer price of r_u is established. The quantity of u_2 sold will be determined as soon as Division 2 commits itself to a quantity of q as in the last deterministic model. Cost of production for Division 1 is given by $C_1(u_1 + u_2)$. Their profit is then given by

$$\Pi_1 = \begin{Bmatrix} m\mu & \text{for } \mu < u_1 \\ mu_1 & \text{for } \mu \geq u_1 \end{Bmatrix} + r_u u_2 - C_1(u_1 + u_2)$$

and the expected profit is

$$E\Pi_1 = \int_0^{u_1} m\mu s(\mu)d\mu + \int_{u_1}^{\infty} mu_1 s(\mu)d\mu + r_u u_2 - C_1(u_1 + u_2)$$

$$= \int_0^{\infty} m\mu s(\mu)d\mu - \int_{u_1}^{\infty} m(\mu - u_1)s(\mu)d\mu + r_u u_2 - C_1(u_1+u_2)$$

$$E\Pi_1 = mE\mu - \int_{u_1}^{\infty} m(\mu - u_1)s(\mu)d\mu + r_u u_2 - C_1(u_1 + u_2). \qquad (1)$$

This last manipulation involved the facts that $E\mu = \int_0^{\infty} \mu s(\mu)d\mu$ and $\int_0^{u_1} m\mu s(\mu)d\mu = \int_0^{\infty} m\mu s(\mu)d\mu - \int_{u_1}^{\infty} m\mu s(\mu)d\mu$. The quantity $E\mu$ represents the expected value of demand for the product u_1. The term under the integral can be regarded as the expected revenue foregone due to shortage.

Market demand for the final product is represented by the random variable ϵ which is always greater than zero and has some known probability distribution given by $w(\epsilon)$. The same circumstances of prior commitment exist for Division 2 as did for Division 1. Division 2's

known costs are given by $r_u u_2$ and $C_2(q)$ where the second term accounts for the production cost of all inputs other than u_2. Profit for the division is

$$\Pi_2 = \begin{Bmatrix} p\epsilon & \text{for } \epsilon < q \\ pq & \text{for } \epsilon \geq q \end{Bmatrix} - C_2(q) - r_u u_2$$

and expected profit is represented by the relation

$$E\Pi_2 = pE\epsilon - \int_q^\infty p(\epsilon - q)w(\epsilon)d\epsilon - C_2(q) - r_u u_2 \qquad (2)$$

which has been derived in the same way as equation (1). For the firm as a whole profit is represented by

$$\Pi = \begin{Bmatrix} m\,\mu & \text{for } \mu < u_1 \\ mu_1 & \text{for } \mu \geq u_1 \end{Bmatrix} + \begin{Bmatrix} p\epsilon & \text{for } \epsilon < q \\ pq & \text{for } \epsilon \geq q \end{Bmatrix} - C_1(u_1 + u_2) - C_2(q)$$

and expected profit by

$$E\Pi = \int_0^{u_1} \int_0^q (m\,\mu + p\,\epsilon) s\,(\mu)\, w\,(\epsilon)\, d\epsilon\, d\,\mu$$

$$+ \int_0^u \int_q^\infty (m\,\mu + pq)\, s\,(\mu)\, w\,(\epsilon)\, d\epsilon\, d\,\mu$$

$$+ \int_{u_1}^\infty \int_0^q (mu_1 + p\epsilon)\, s\,(\mu)\, w\,(\epsilon)\, d\epsilon\, d\,\mu$$

$$+ \int_{u_1}^\infty \int_q^\infty (mu_1 + pq)\, s(\mu)\, w(\epsilon)\, d\epsilon\, d\,\mu$$

$$- C_1(u_1 + u_2) - C_2(q).$$

By assuming the demands and demand distributions for the intermediate and final products are independent, the above equation can be simplified to

$$E\Pi = mE\,\mu + pE\,\mu + pE\,\epsilon - \int_{u_1}^{\infty} m(\mu - u_1)s(\mu)d\mu - \int_{q}^{\infty} p(\epsilon - q)w(\epsilon)d\epsilon$$

$$- C_1(u_1 + u_2) - C_2(q) \tag{3}$$

If the managers of the firm and its divisions have linear risk preferences in terms of profit and their sole goal is profit maximization, then maximization of equations (1), (2), and (3) becomes the objective. The decision variables for the firm, Division 1, and Division 2 are u_1 and q, u_1, and q respectively. Assuming further that external market prices are constant for any output levels set by the firm or all firms, the first order conditions for profit maximization can be found by taking partial derivatives with respect to the decision variables and equating the results to zero. This gives

Division 1:

$$\frac{\partial E\Pi_1}{\partial u_1} = \int_{u_1}^{\infty} ms\,(\mu)d\,\mu - \frac{\partial C_1(u_1 + u_2)}{\partial u_1} = 0 \tag{4}$$

Division 2:

$$\frac{\partial E\Pi_2}{\partial q} = \int_{q}^{\infty} pw\,(\epsilon)d\,\epsilon - \frac{\partial C_2(q)}{\partial q} - \frac{\partial r_u u_2}{\partial u_2} \cdot \frac{\partial u_2}{\partial q} = 0 \tag{5}$$

Firm:

$$\frac{\partial E\Pi}{\partial u_1} = \int_{u_1}^{\infty} ms(\mu)d\,\mu - \frac{\partial C_1(u_1 + u_2)}{\partial u_1} = 0 \tag{6}$$

$$\frac{\partial E\Pi}{\partial q} = \int_q^\infty pw(\epsilon)d\epsilon - \frac{\partial C_2(q)}{\partial q} - \frac{\partial C_1(u_1 + u_2)}{\partial (u_1 + u_2)} \cdot \frac{\partial u_2}{\partial q} = 0 \tag{7}$$

In order to insure that maximizing conditions for the divisions are the same as the maximizing conditions for the firm it is necessary that

$$\frac{\partial (r_u u_2)}{\partial u_2} \cdot \frac{\partial u_2}{\partial q} = \frac{\partial C_1(u_1 + u_2)}{\partial u_2} \cdot \frac{\partial u_2}{\partial q} \tag{8}$$

or

$$r_u = \frac{C_1(u_1 + u_2) + k}{u_2} \tag{9}$$

where k represents the constant of integration. Therefore, the optimal transfer price in this case is the same as it is in the deterministic situation.

<u>Holding Cost</u>

Thus far the model has dealt only with perishable products and those that must be sold in the same period they are manufactured. There are, however, many commodities in which it is possible to save or store unsold output for sale at a later time. When this occurs, it is not accomplished without expense, and the outlay involved is usually referred to as a holding or inventory cost.

There are different methods available for calculating holding costs. One can either assume that all sales occur at the beginning of the period and the excess is stored, or that sales occur continuously throughout the period and that some average inventory is stored. The

method of this model will be the former and the holding cost for a period will be given by a function of the excess of production over demand. If the unit storage cost is h_i where $i = u$ or q, then holding costs for the first and second divisions can be represented by

$$H(u_1) = \begin{Bmatrix} h_u(u_1 - \mu) & \text{for } u_1 > \mu \\ 0 & \text{otherwise} \end{Bmatrix}$$

and

$$H(q) = \begin{Bmatrix} h_q(q - \epsilon) & \text{for } q > \epsilon \\ 0 & \text{otherwise} \end{Bmatrix}$$

The firm as a whole is confronted by four possible combinations of market excesses and shortages, which are

Case 1: $\mu < u, \epsilon < q$

Case 2: $\mu < u, \epsilon > q$

Case 3: $\mu > u, \epsilon < q$

Case 4: $\mu > u, \epsilon > q$

In the situation represented by the first case, both divisions have produced more than their respective demands; consequently, both incur holding costs. For the firm these costs will be additive. Case 4 illustrates the occasion in which demands exceed supplies and holding costs for both divisions and the firm are zero.

The situation for the second and third cases, however, becomes somewhat more complex. Each is characterized by an excess of production in one division and a shortage in the other. For some products, the excess commodity or some multiple thereof might be used to satisfy that shortage. In others, they will not be substitutable in any form.

Mathematical formulation of the former is quite lengthy, while that of the latter is direct and relatively simple.

For this model, assume that it is not possible to utilize one division's unsold output to satisfy unexpected demand for the other. In other words, the intermediate product is not acceptable in Division 2's market, nor can it be extracted from the final product to satisfy Division 1's demands. In addition to being realistic, this simplifies the following calculations considerably. With this assumption, expected holding costs for each of the divisions is given by

$$EH(u_1) = \int_0^{u_1} h_u(u_1 - \mu)\, s(\mu)\, d\mu \tag{10}$$

$$EH(q) = \int_0^{q} h_q(q - \epsilon)\, w(\epsilon)\, d\epsilon \tag{11}$$

and the expected holding cost for the firm is their sum.

To include holding costs in the basic model it is necessary to subtract the cost given by equation (10) from equation (1), equation (11) from equation (2), and equations (10) and (11) from equation (3). If the partial derivatives with respect to the appropriate decision variables of these newly formed equations are set identically to zero, the condition for simultaneous optimization is again found to be given by equation (8).

If the conditions of cases 2 and 3 hold and it is assumed that there is some substitutability of the excess product for the one in which a shortage exists, the mathematical formulations of the model become quite lengthy. It is likely that the transfer price in such a

situation will contain the random variables of demand, their distributions, or both.

Lack of an Intermediate Market

The Enzer deterministic transfer pricing model discussed both cases in which the intermediate product had and did not have an external market. So far the stochastic model has considered only the case in which there is an intermediate market. If that market is assumed nonexistent all terms in equations (1), (2), and (3) containing a factor of m will vanish and all references to u_1 will drop out. As Division 1 is only producing u_2 for sale to Division 2, it can be shown that equation (8) will reduce to

$$\frac{\partial (r_u u_2)}{\partial u_2} = \frac{\partial C_1(u_2)}{\partial u_2} \qquad (8')$$

Hence, the optimal transfer price remains some sort of an average cost, which is the same result obtained in the deterministic case.

Imperfect Market Competition

The formulation of the model thus far has been under the implicit assumption that market prices for the product were not within the control of the firm. If that assumption is altered to an imperfectly competitive market where $m = m(u_1)$ and $p = p(q)$, there will be changes in the form of equations (4), (5), (6), and (7) due to the fact that $\partial m(u_1)/\partial u_1$ and $\partial p(q)/\partial q$ are not necessarily m and p respectively. The changes that occur in (4) and (5) will also occur in (6) and (7) resulting in equation (8) still representing the necessary requirement for simultaneously maximizing conditions. The optimal transfer price

will therefore remain an average cost in the imperfectly competitive market situation.

Monopolistic Market

If the firm is a monopolist the basic model is altered to the extent that the decision variables become production quantities and market prices. Mills (Ref 20:116-130) and Zabel (Ref 27:205-218) have both examined monopolistic markets under conditions of uncertainty but with different assumptions about the form of the uncertainty.

Additive Stochastic Term. Mills assumed that the external market demand is a function of the respective market price and some additive random term. The random variable is assumed to be independent of the price associated with its commodity, to have a zero expectation, and to have a known distribution. In the case of the basic model, for example, the random variables might be x and y and their associated distributions might be f(x) and f(y). Then

$$\mu = d(m,y) = d(m) + y$$
$$\epsilon = d(p,x) = d(p) + x$$

where $d(\cdot)$ represents the deterministic demand as a function of price. The effect of this type of random variable will be to cause shifts in the demand function parallel to itself in the quantity direction (Ref 20:118).

If the additive assumption for the stochastic demand is used, it becomes necessary to perform a change of variable with respect to the integration argument. After such an operation is performed,

equation (1), for example, becomes

$$E\,\Pi_1 = md(m) + m\int_{u_1-d(m)}^{\infty} yh(y)dy - m(u_1 - d(m))\int_{u_1-d(m)}^{\infty} h(y)dy$$

$$+ r_u u_2 - C_1(u_1 + u_2) \qquad (1')$$

Similar results can be derived for Division 2 and the firm. If partial derivatives of these equations are taken with respect to m and u_1 for Division 1, p and q for Division 2, and m, u_1, p, and q for the firm, the necessary condition for simultaneous optimization is again found to be given by equation (8). Thus, under the additive random variable assumption the optimal transfer price is still some form of an average cost.

Note that the introduction of uncertainty as an additive random variable in the basic model produces the same optimal transfer price that was found in the deterministic model. This should not be interpreted as implying that all optimizing conditions will be the same for the stochastic and deterministic models. The objective of the Mills model was to determine optimal market prices and quantities, and he found that price and output should be such that riskless marginal revenue is somewhat less than, or at most equal to, marginal cost (Ref 19:381). The same results can be derived from the present model by solving for the external market prices and production quantities. This differs from the typical deterministic result of equating marginal revenue to marginal cost.

Multiplicative Stochastic Factor. Zabel's model assumed that demand is a multiplicative function of some random variable such that $\mu = zd(m)$. The random variable in this case is denoted by z which is always non-negative. The expected value of z is equal to one and it has a known density function $g(z)$. As in the Mills model, the factor $d(m)$ represents the deterministic or expected value of demand. Deterministic demand is a function of m and it is required that $\mu = 0$ whenever $d(m) = 0$. Zabel claims that this form of monopolistic uncertainty is more realistic than the additive function used by Mills because demand under the multiplicative form can never be negative (Ref 27:207).

Using the multiplicative random variable in the basic model again requires a change of variable. If this is accomplished, equation (1) becomes

$$E\,\Pi_1 = md(m) + \int_{u_1/d(m)}^{\infty} mu_1 g(z)dz - \int_{u_1/d(m)}^{\infty} mzd(m)g(z)dz + r_u u_2 - C_1(u_1 + u_2) \qquad (1'')$$

Once again similar equations can be derived for Division 2 and the firm, but the final requirement for simultaneous optimization remains equation (8).

Stochastic Costs

All model formulations thus far have dealt with variations of stochastic demand. That assumption will now be dropped, demands will be considered deterministic, and elements of stochastic cost will be examined.

Final Product Costs. For the first variation assume that only costs of inputs to q other than u are stochastic in nature. These costs were designated as $C_2(q)$ in the deterministic model, but will be designated as K_2 in the stochastic cost model. If K_2 is a random variable with :nown distribution of $f(k_2)$, then the expected cost of inputs to q other than u is $EK_2 = \int_0^\infty k_2 f(k_2)dk_2$. The expected profit equations then become

$$E\,\Pi_1 = mu_1 + r_u u_2 - C_1(u_1 + u_2)$$

$$E\,\Pi_2 = pq - r_u u_2 - EK_2$$

$$E\,\Pi = mu_1 + pq - C_1(u_1 + u_2) - EK_2$$

If the partial derivatives of these equations with respect to the appropriate decision variables are examined, the results show that for simultaneous optimality equation (8) from the stochastic demand model must hold. In this case the optimal transfer price remains an average cost. Note that if the additive or multiplicative random variable form of uncertainty replaces K_2 in the above equations, the results for the optimal transfer price do not change.

Intermediate Product Costs. Assume now that the cost of producing the intermediate product is a random variable denoted as K_1 which has some known probability distribution given by $f(k_1)$. Again the expected cost of the intermediate product is given by $EK_1 = \int_0^\infty k_1 f(k)dk_1$. This value can be substituted into the expected profit equations and partial differentiation with respect to the various decision variables will give the first order conditions for optimality. These are

$$\frac{\partial mu_1}{\partial u_1} - \frac{\partial EK_1}{\partial u_1} = 0$$

$$\frac{\partial pq}{\partial q} - \frac{\partial r_u u_2}{\partial q} - C_2'(q) = 0$$

$$\frac{\partial mu}{\partial u_1} - \frac{\partial EK_1}{\partial u_1} = 0$$

$$\frac{\partial pq}{\partial q} - C_2'(q) - \frac{\partial EK_1}{\partial q} = 0.$$

For simultaneous optimality to hold, it is necessary that

$$\frac{\partial r_u u_2}{\partial q} = \frac{\partial EK_1}{\partial q} .$$

If this equation is integrated and then divided by u_2, the optimal transfer price becomes some type of an average expected cost.

The form of this last value for transfer price can be simplified considerably through the use of the additive and multiplicative random variable assumptions. Suppose, for example, that the stochastic cost K_1 is a function of the quantity of u produced and some independent random variable denoted by y such that $K_1 = C_1(u,y)$. Independency permits this to be written as $K_1 = C_1(u) + y$ where the first term is the deterministic cost. Assuming further that the expected value of y is zero and it is distributed as k(y), a change of variable permits the equation for Division 1 to be rewritten as

$$E\Pi_1 = mu_1 + r_u u_2 - \int_0^\infty [C_1(u) + y]\, h\,(y)\, dy$$

$$= mu_1 + r_u u_2 - C_1(u).$$

Performing analogous operations on the cost of u for the firm results in the previously determined first order optimality condition again being equation (8). Hence, in this case the optimal transfer price is still some form of an average cost as given by (9).

Suppose now that the stochastic cost is a function of some multiplicative random variable such that $K_1 = zC_1(u)$ where $C_1(u)$ is again the deterministic cost of u and z is a random variable with an expectation of one, always non-negative, and distributed as g(z). If the required change of variable is again performed, the expected profit for the first division will be given by

$$E\Pi_1 = mu_1 + r_u u_2 - \int_0^\infty zC_1(u)\, g\,(z)\, dz$$

$$= mu_1 + r_u u_2 - C_1(u)$$

Similar operations can be performed on the equation of the firm and the first order conditions derived via the partial derivatives. Once again simultaneous optimality requires that equation (8) be satisfied and the optimal transfer price remains an average cost as in (9).

Another possible variation of stochastic costs might occur in the form of a budget constraint. Suppose for example that the costs previously denoted by $C_1(u_1 + u_2)$ and $C_2(q)$ are deterministic. The firm has at its disposal some fixed amount of monetary resources

denoted by $\bar{C}$ which is assumed to be an insufficient amount of cash to finance the optimal production level. The firm has the option of borrowing additional funds but cannot be certain what quantities will be available due to demands for financing and prevailing interest rates. If, however, the amount of cash obtainable is denoted by K which has a known probability distribution denoted by f(k), then the firm's expected constraint is

$$C_1(u_1 + u_2) + C_2(q) \leq \bar{C} + \int_0^\infty kf(k)dk$$

This results in expected profit equations of the form

$$E\Pi_1 = mu_1 + r_u u_2 - C_1(u_1 + u_2)$$

$$E\Pi_2 = pq - C_2(q) - r_u u_2$$

$$E\Pi = mu_1 + pq - C_1(u_1 + u_2) - C_2(q)$$

$$- \lambda\left[C_1(u_1 + u_2) + C_2(q) - \bar{C} - \int_0^\infty kf(k)dk\right]$$

where λ is a Lagrangian multiplier. The resulting first order conditions are

Division 1: $$\frac{\partial mu_1}{\partial u_1} - \frac{\partial C_1(u_1 + u_2)}{\partial u_1} = 0$$

Division 2: $$\frac{\partial pq}{\partial q} - \frac{\partial C_2(q)}{\partial q} - \frac{\partial r_u u_2}{\partial q} = 0$$

Firm: $$\frac{\partial mu_1}{\partial u_1} - \frac{\partial C_1(u_1 + u_2)}{\partial u_1}[1 + \lambda] = 0 .$$

$$\frac{\partial pq}{\partial q} - \frac{\partial C_2(q)}{\partial q}[1 + \lambda] - \frac{\partial C_1(u_1 + u_2)}{\partial q}[1 + \lambda] = 0$$

$$C_1(u_1 + u_2) + C_2(q) - \overline{C} - \int_0^\infty kf(k)dk = 0$$

To produce simultaneous optimality in this case it is necessary for C_1 and C_2 for Divisions 1 and 2 to be adjusted to $[1 + \lambda]C_1$ and $[1 + \lambda]C_2$. With this variation of stochastic costs, the optimal transfer price is found to be

$$r_u u_2 = (1 + \lambda) \int \frac{dC_1(u_1 + u_2)}{du} du_2 + c$$

or

$$r_u = (1 + \lambda) \frac{C_1(u_1 + u_2) + c}{u_2} ,$$

where c is the constant of integration. Hence, the optimal transfer price is an average cost adjusted by $(1 + \lambda)$.

V. Application of the Model to DOD

An Application

The basic model and its variations presented thus far have been based on a firm in the private sector in order to facilitate the derivation of theoretical implications. It is instructive to examine now how that model might apply to the defense establishment, and in particular to the situation that occurred at AEDC.

The basic model utilized a firm with two divisions. For the military model, Division 1 will be supplanted by AEDC, Division 2 by a System Program Office (SPO), and the firm by Air Force Systems Command (AFSC). The outputs of these new divisions are a T&E capability transferred to the SPO as a service and a weapon system transferred through AFSC to the major air commands. In addition, the final product can possibly be marketed to other services and foreign countries. The "firm," AFSC, is ultimately responsible for both of its "divisions" and their outputs.

For Air Force purposes AEDC is not a profit motivated organization. The commander of AEDC or any other T&E facility is motivated to stay within his budget, to meet users' schedules and obtain the required data, and to maintain and improve a viable test support capability for his continually changing list of users. In a similar manner, the SPO commander lacks profit orientation, but he does have a concern with getting the test support where, when, and how he needs it. His motivations are to stay within the budget and to meet system design and performance criteria (Ref 9:21-22). AFSC shares these and

other basic motivations. Consequently, it is reasonable to assume that a simplified objective for all three organizations might well be performance constrained cost minimization. The basic model must now be examined for application to this situation.

Equations (1), (2), and (3) of the model presented can be written in general form as

$$E(\text{Profit}) = \text{Market Price} \times E(\text{Demand}) - E(\text{Cost})$$

If the decision variables for each division are their respective quantities of output, then maximization of expected profit and minimization of expected cost produce identical results. Because of this, the model can be altered to reflect the DOD environment.

With the objective of performance constrained cost minimization, it is first necessary that performance criteria be explicitly spelled out. Subsequent divisional and managerial evaluations must reflect achievements vis-a-vis performance goals. The other facet of the objective, cost minimization, can be formulated by elimination of the revenue terms from equations (1), (2), and (3). The resulting cost equations contain the integration terms that were referred to as expected revenue foregone in the basic model. In the case of the cost minimization model they are more appropriately regarded as the expected cost of supplying output at a later time if a shortage does occur. For brevity, both will be referred to as opportunity costs.

Opportunity costs in the basic model were regarded as identical for the firm and its divisions. For AFSC and AEDC this may not be the case. It is reasonable to consider that failure to satisfy demand might cause consternation among the management at AEDC which does not necessarily correspond to the attitudes at AFSC. Personnel at AEDC

have their job security and professional status to protect, both of which can be lost by failure to meet demand. But AFSC can conceivably achieve similar test results by using another facility, either public or private. Consequently, the opportunity costs for failure to satisfy the intermediate market demand will be assumed dissimilar with m_1 and m_F representing that cost for AEDC and AFSC respectively. As AFSC typically assigns a specific task to a single SPO, the opportunity cost for the final product will be considered identical for both the SPO and AFSC.

The T&E Funding Policy Study headed by Berquist stated that many management and resource constraints acted to preclude optimum testing. Some of those listed were lack of funds, multiple management echelons, civilian personnel ceilings, competing requirements, and the inherent nature of testing (Ref 9:35). A possible approach for AFSC might be insertion of some of these constraints into the model followed by the use of simulations to explore the directions transfer pricing should theoretically take.

For example, consider such constraints as lack of funds, multiple management echelons, and the inherent nature of testing. Lack of funds is almost always a real life constraint as DOD budgets are limited. The reference to multiple management echelons is another way of saying that programs and resources flow through several command echelons each of which may impose regulations and limitations on their use. An example of this type of constraint would be insistence by higher authorities that a specific testing capability should not be permitted to fall below a certain level. The last of these, the inherent nature of testing, refers to the fact that the amount of

testing required to produce a final product is not generally known in advance.

Recall that an explicit assumption of the basic stochastic and deterministic models was that the production function relationship between the intermediate and final products was known. At first glance, the inherent nature of the testing constraint appears to have significant impact on that assumption. However, the production function will appear in exactly the same form, whatever that may be, in the equations of both the SPO and AFSC. Consequently, this difference may alter the formulation of the model, but the calculus of the two equations will be identical with respect to those terms. Consider now applications of the budget and minimum output constraints to the revised model.

Suppose that a specific testing facility in question is a high altitude hypersonic gas dynamic wind tunnel located at AEDC. Demand for the facility for a forthcoming period might come only from a single SPO charged with the development of high speed reentry vehicles. Hence, the intermediate product, high speed tunnel test capability, might not currently have an external market and could be designated as u. The final product, q, could be a reentry vehicle with certain performance characteristics and an indeterminate demand. Costs of the two products are reasonably well fixed once quantities are established due to the fact that personnel are salaried and materials are contracted. These may be represented by C(u) and C(q). If the funds appropriated for these activities are less than an optimal amount, say $\bar{C}$, then the budget constraint can be written as $C(u) + C(q) \leq \bar{C}$. The fact that AFSC does not want the output of u to

be less than a certain amount, say u^o, can be expressed as $u \leq u^o$. This last term can be rewritten as $u = u^o + s^2$ where s^2 is a slack variable introduced to ensure that u cannot be less than u^o.

If these constraints and the other differences cited are introduced, the equations for AEDC, the SPO, and AFSC become

$$EC_1 = \int_u^\infty m_1(\mu - u)\ s(\mu)\ d\mu + C(u)$$

$$EC_1 = \int_q^\infty p(\epsilon - q)\ w(\epsilon)\ d\epsilon + C(q) + r_u u$$

$$EC_F = \int_u^\infty m_F(\mu - u)\ s(\mu)\ d\mu + \int_q^\infty p(\epsilon - q)\ w(\epsilon)\ d\epsilon + C(u)$$

$$+ C(q) + \lambda[u - u^o - s^2] + \theta[C(u) + C(q) - C].$$

Taking the partial derivatives with respect to the respective output quantity decision variables results in

$$\frac{\partial EC_1}{\partial\ u} = -m_1 \int_u^\infty s(\mu)\ d\mu + C'(u) = 0$$

$$\frac{\partial EC_2}{\partial\ q} = -p \int_q^\infty w(\epsilon)\ d\epsilon + C'(q) + \frac{\partial r_u u}{\partial\ q} = 0$$

$$\frac{\partial EC_F}{\partial\ u} = -m_F \int_u^\infty s(\mu)\ d\mu + C'(u)\ [1 + \theta] + \lambda = 0$$

$$\frac{\partial EC_F}{\partial\ q} = -p \int_q^\infty w(\epsilon)\ d\epsilon + C'(q)\ [1 + \theta] + \frac{\partial C(u)}{\partial\ q}[1+\theta] + \frac{\partial u}{\partial q} = 0.$$

In order to have conditions of optimality, the opportunity cost of AEDC given by m_1 times the integral must be adjusted by the factor m_F/m_1, and the costs C(u) and C(q) incurred by AEDC and the SPO must both be altered by the factor $[1 + \theta]$. Finally, to get at the optimal transfer price the following relationship must hold

$$\frac{\partial r_u u}{\partial u} \cdot \frac{\partial u}{\partial q} = \frac{\partial C(u)}{\partial u} \cdot \frac{\partial u}{\partial q} [1 + \theta] + \lambda \frac{\partial u}{\partial q}$$

which results in

$$r_u = \frac{(1 + \theta)\, C(u) + \int \lambda du + c}{u}$$

where c is the integration constant. The optimal transfer price is again some sort of an average cost. In order to get at the specific nature of that cost, definite values and equations would have to be substituted and perhaps iterated to determine a k that coincided with the multiple objectives of AFSC.

Other variations with different constraints should be possible and could serve as an aid to determining transfer prices under a variety of situations. This should not be attempted, however, until some difficulties inherent in the application of this model to the public sector are examined.

Problem Areas

A discussion on decentralization in DOD is beyond the scope of this study, but some observations on decentralization in the T&E community and its users should be made. It seems, for example, that motivating managerial activity with a performance constrained cost

minimization objective criterion could create severe problems. Weapons system development lead times frequently exceed normal military duty tours, and it is not infrequent that performance evaluations are completed only after introduction into the operational inventory. Hence, the manager is motivated by two performance criteria that do not always occur in the same time frame. Cost information will be readily available during and at the end of each cost accounting period, while performance information may not be known until after the manager has been reassigned. Under these conditions it is reasonable to assume that some managers will be highly motivated by the short term considerations of their achievement evaluations at the expense of long range system performance.

To offset this counter productive motivation there must be some form of effective monitoring of end-service achievements. The method of achieving this goal without encumbering the participants with detailed operating rules designed to ensure performance could be a monumental task in itself (Ref 10:486).

Another important consideration is the amount of choice open to the various divisions in purchasing inputs and services. Detailed regulations from above specifying exactly how things are to be done are incompatible with lower-level discovery and the adoption of efficient ways to combine resources in operations (Ref 10:485). Furthermore, the very nature of T&E facilities dictates that the amount of choice available will vary from one test situation to another. In some instances activities will have complementary facilities and limited regions of overlapping capabilities. In other, more commonly tested areas alternatives may exist on an interservice,

intraservice, or corporate basis but it may not be permissible to choose one of them.

Still another problem exists as a result of the nature of the test facility. Usually they are characterized by large investments, limited usage, and high fixed operating expenses. The human and physical resources required take a fairly long time to develop and usually cannot be acquired or implemented on short notice. Consequently, it is very difficult to make short term capability adjustments that correspond to short term demand variations.

Finally, the Berquist study noted that there were a myriad of funding policies in use at the 26 major facilities and that standardization is highly desirable. However, it seems appropriate to ask whether or not the system being imposed is de facto or superficial decentralization. In either case it must be judged on the basis of its differential costs and its improved benefits as compared to the system it replaces. If, however, it is superficial, then attempts to improve resource allocation and managerial performance through the transfer price mechanism are destined to meet with frustration.

VI. Conclusions

One of the basic intentions of this study was to compare the results of an optimal transfer pricing rule under deterministic and stochastic conditions. Consequently, the objectives of the firm and its divisions were assumed to be profit maximization and expected profit maximization respectively. This is obviously a simplification of the real world as corporations generally have numerous objectives. Some of these might be sales, growth, survival, social responsibility, return on investment, and owner and employee welfare. Hence, this model falls well short of being a perfect predictor of what decisions should be made, but it is hopefully an indicator of some relevant implications that management should consider in the decision process.

The approach of this study has assumed that market demands and production costs are unknown but come from some known or presumed known probability distributions. Application of the techniques of stochastic inventory control and use of expected value as a decision criterion have shown that for a variety of nondeterministic situations the optimal transfer price has consistently proven to be some form of an average cost. Although the exact form of this cost is not immediately obtainable from the foregoing derivations, insertion of precise equation forms into the model and the use of iterative processes should give approximations that will provide decision makers with the direction of optimal transfer prices.

The stochastic model was based on and provides results similar to that of a model developed by Herman Enzer for examination of

transfer pricing under a variety of static, deterministic conditions. Enzer's model was developed as a result of shortcomings in the marginal cost transfer pricing rule theorized by Jack Hirshleifer. Those shortcomings appeared earlier in this study and were: (1) an inability of users to develop the required marginal cost and marginal revenue data; (2) a necessary sacrifice of divisional autonomy; (3) a possibility of one division being forced to operate at a loss; and (4) the limited utility of marginal decisions. A brief comparison of the effects of the average cost pricing rule on these shortcomings follows:

1. At first glance cost information appears readily available. However, the optimal transfer price derived in theory contained an integration constant that could prove troublesome to accounting and bookkeeping personnel. Specific situations would have to be applied to the model to gain insight into what information is necessary and available.

2. The Hirshleifer model required a specific constraint in order to prevent suboptimization. As of this writing, there is no known constraint necessary to ensure optimization.

3. The average cost model will provide the same results if each division acts either to maximize profit or minimize cost. Hence, all responsibility centers will not have to operate as profit centers. By operating as cost centers, then, service divisions will not be measured by the same criterion as distribution divisions.

4. From all apparent indications the average cost rule will also be of more benefit in marginal operating decisions. It is possible, however, that average cost information may facilitate the accumulation of the total magnitudes necessary for capital decisions.

On the whole, then, the average cost rule appears to make up for some of the deficiencies of the marginal cost rule. Whether or not other problem areas will develop with the average cost optimal transfer pricing rule remains to be seen.

The stochastic model presented can probably be altered and applied to government sectors where decentralization and interdivisional

exchange are appropriate. It is likely, however, that after all the constraints peculiar to the public sector have been considered, the optimal transfer price will still turn out to be some form of an average cost. The results of the simplified formulation of the AEDC situation tend to confirm this supposition. Similar to the other models presented, the average cost derived in that case contained an integration constant. That constant can probably be set such that the optimal transfer price will be an average variable or average total cost. This theoretically derived result tends to substantiate the empirically derived alternatives suggested by the Berquist Study Group. That study recommended transfer prices based on direct cost or direct plus indirect cost.

Finally, it should be recalled that the most important aspect in application of this or any other transfer pricing model is the issue of decentralization. If decentralization is a reality, the average cost rule should prove successful. But if it is an accounting procedure, improvements in resource allocation will probably be in spite of rather than because of the transfer pricing rule implemented.

Bibliography

1. __________. Air Staff Briefing presented to SAF Dr. Semans, 1971.

2. __________. Service Funding for Research and Development, A Resource Management System Case Study (70-5). Washington: Office of the Assistant Secretary of Defense (Comptroller), December 1970.

3. AFR 170-1. Resource Management System (RMS). Washington: Department of the Air Force, 20 May 1969.

4. Arrow, K. J. "Control in Large Organizations." Management Science, Vol. 10 No. 3:397-408 (April 1964).

5. Arrow, K. J., Samuel Karlin, and Herbert Scarf. Studies in the Mathematical Theory of Inventory and Production. Stanford, California: Stanford University Press, 1958.

6. Anthony, R. N. Management Accounting: Text and Cases. Homewood, Illinois: Richard D. Irwin, Inc., 1970.

7. Bailey, Martin J. "Defense Decentralization Through Internal Prices" in Defense Management, edited by Stephen Enke. Englewood Cliffs, New Jersey: Prentice-Hall Inc., 1967.

8. Berkwitt, George J. "Do Profit Centers Really Work." Dunn's Review, Vol 93, No. 5:29-117 (May 1969).

9. Berquist, George W., et al. "A Study of Funding Policy for Major Test and Evaluation Support Activities." Washington: Department of Defense, April 1970.

10. Breckner, N. V. "Government Efficiency and the Military "Buyer-Seller" Device." The Journal of Political Economy, 68:469-486 (October 1960).

11. Dean, Joel. "Decentralization and Intracompany Pricing." Harvard Business Review, XXXIII:65-74 (July-August 1955).

12. Deardon, John. "The Case of the Disrupting Divisions." Harvard Business Review, XLII:159-178 (May-June 1964).

13. Deardon, John and Bruce Henderson. "New System for Divisional Control." Harvard Business Review, XLIV:144-160 (September-October 1966).

14. Enzer, Herman. "The Static Theory of Transfer Pricing." Wright-Patterson Air Force Base, Ohio: AFIT School of Engineering, January 1973.

15. Gordon, Myron J. "The Use of Administered Price Systems to Control Large Organizations" in Management Controls, New Directions in Basic Research, edited by C. P. Bonini, R. K. Jaedicke, and H. M. Wagner. New York: McGraw-Hill Inc., 1963.

16. Hillier, Frederick S. and Gerald J. Leiberman. Introduction to Operations Research. San Francisco: Holden-Day, Inc., 1967.

17. Hirshleifer, Jack. "On the Economics of Transfer Pricing." The Journal of Business, XXIX:172-184 (April 1956).

18. Hitch, C. J. and R. N. McKean. The Economics of Defense in the Nuclear Age. Boston: Harvard University Press, 1960.

19. Horowitz, Ira. Decision Making and the Theory of the Firm. Chicago: Holt, Rhinehart and Winston, Inc., 1970.

20. Mills, Edwin S. "Uncertainty and Price Theory." Quarterly Journal of Economics, Vol. 73, No. 1:116-130 (February 1959).

21. Niskanen, William A. "The Defense Resource Allocation Problem" in Defense Management, edited by Stephen Enke. Englewood Cliffs, New Jersey: Prentice-Hall Inc., 1967.

22. Packard, David. Deputy Secretary of Defense Memorandum for the Secretaries of the Military Departments and Others, Washington, D. C., 20 October 1971.

23. Schlaifer, Robert. Probability and Statistics for Business Decisions. New York: McGraw-Hill Inc., 1959.

24. Schnurr, D. V. Associate Director of Budget Memorandum for the Record, Washington, D. C., 15 December 1969.

25. Selman, Jerome H. N., and Victor Selman. "Operations Research for Risk Analysis Evaluation."

26. Shillingham, Gordon. Cost Accounting: Analysis and Control. Homewood, Illinois: Richard D. Irwin, Inc., 1967.

27. Zabel, E. "Monopoly and Uncertainty." Review of Economic Studies, Vol XXXVII (2), No. 110:205-219 (April 1970).

28. Zannetos, Zenon S. "On the Theory of Divisional Structures: Some Aspects of Centralization and Decentralization of Control and Decision Making." Management Science, Vol. 12, No. 4:B-49-B-68 (December 1965).

VITA

James Dennis Bain [redacted] He graduated from high school in Benton, Illinois in 1960, attended the University of Illinois and then the United States Air Force Academy from which he received a Bachelor of Science degree and a commission in the USAF in 1965. He received his wings in September 1966, and subsequently served as a C-130B pilot in Southeast Asia and as a rescue HC-130H/N pilot in USAFE. In September of 1971 he was assigned to the Air Force Institute of Technology for study leading to a Master of Science degree in Systems Analysis.

[redacted]

This thesis was typed by Mrs. Frances Jarnagin.